Table 9.8 SAMPLING FREQUENCIES: DEMONSTRATION PROJECT, CASE II

BUDGET 25000.00

SOURCE	MIN NO. SAMPLES REQUIRED	MAX NO. SAMPLES ALLOWED	TIMES SAMPLED	RESOURCES USED	COST OF UNDETECTED VIOLATIONS
1	0	3	3	1764.00	.92752
2	0	3	1	591.00	.25123
3	0	3	0	.00	.00061
4	0	3	3	1713.00	.46674
6	0	3	2	1152.00	.35770
7	0	3	2	1132.00	.04382
8	0	3	1	603.50	.00029
9	0	3	2	1166.00	.00930
10	0	3	2	1136.00	.03195
11	0	3	0	.00	2.09945
12	0	3	2	1136.00	.16065
13	0	3	3	1644.00	1.45490
14	0	3	0	.00	2.26727
15	0	3	1	535.50	.00204
16	0	3	0	.00	4.12032
17	0	3	2	1887.00	.39854
18	0	3	1	565.00	.19847
19	0	3	0	.00	3.82261
20	0	3	1	543.00	.24557
22	0	3	2	1126.00	.29159
23	0	3	3	1680.00	.28319
24	0	3	0	.00	.92344
25	0	3	2	1100.00	.04121
26	0	3	1	563.00	.22996
27	0	3	1	560.00	.00000
28	0	3	3	1689.00	1.44380
29	0	3	3	1650.00	3.28924
30	0	3	1	550.00	.19528

TOTAL RESOURCES USED 24486.00
FINAL COST OF UNDETECTED VIOLATIONS 23.55669

Case III

For Case III, it is assumed that sampling from Case II has been inter-rupted in the middle of a monitoring period. It is expected that Source 14 or Source 19 is contributing to poor water quality. From Table 9.8, it is seen that neither of these sources would normally be sampled during this monitoring period.

Table 9.9 shows the number of times the sources were assumed sampled before the interrupt and the optimal sampling frequencies after the interrupt. Case III has shown how the priority procedure can be used to respond to ambient monitoring reports.

Preliminary Performance Comparison

The performance of the Resource Allocation Program will be compared with a simpler procedure that assigns sampling frequencies on the basis of flow. The latter procedure, called the Allocation by Flow procedure, assigns one sample to all the sources and then assigns the remaining samples, within the budget, to the sources with largest flow.

The monitoring period used for this comparison will be the one corres-ponding to Case II, (i.e., months 19 through 24) where the sampling frequencies were based on data from months 1 through 18.

The performance criteria are (i) the observed "cost" of undetected violations and (ii) the observed number of violators. These criteria are observed values calculated for 14 sources for a month picked at random from the monitoring period.*

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*The number of sources considered for this comparison were reduced to 14 to reduce the amount of data handling required.

Table 9.9 SAMPLING FREQUENCIES BEFORE AND AFTER INTERRUPT:
DEMONSTRATION PROJECT, CASE III.

Source	Times sampled before interrupt	Times sampled after interrupt
1	2	0
2	1	0
3	0	0
4	1	1
6	1	0
7	2	0
8	0	1
9	0	2
10	1	0
11	0	1
12	1	1
13	2	0
14	0	2
15	0	1
16	0	0
17	0	2
18	0	1
19	0	3
20	1	0
22	1	1
23	2	1
24	0	0
25	0	1
26	1	0
27	0	1
28	1	2
29	2	1
30	1	0

The observed "cost" of undetected violations for one month is

$$\bar{c} = \sum_{\text{sources}} \bar{c}_i \ \bar{p}_i^{s_i} \qquad\qquad (9.1)$$

where

$$\bar{c}_i = \text{average damage due to source } i$$

$$= \frac{1}{M} \sum_{k=1}^{M} (\text{observed damage on day } k)$$

$$1 - \bar{p}_i = \text{observed frequency of violation of source } i$$

$$= (\text{number of days in violation}) \div M$$

$$s_i = \text{sampling frequency for source } i.$$

and where M is the number of observed values of the effluent in the month. The observed damage on day k is

$$\max_{\substack{\text{constituents} \\ j}} \{d_j \ (\overline{CO}_{ij}(k))\} \qquad\qquad (9.2)$$

where d_j is the damage function for constituent j and $\overline{CO}_{ij}(k)$ is the concentration of constituent j downstream from source i based on the observed effluent value for constituent j on day k. (Note that the assumed upstream concentration and stream parameters are the same as were used in the priority procedure to determine the sampling frequencies.) The observed number of violators in a month is simply

$$\bar{V} = \sum_{\text{Sources}} (1 - \bar{p}_i^{s_i}) \qquad\qquad (9.3)$$

Table 9.10 shows the observed frequency of violation, $1-\bar{p}_i$, and the average damage, $\bar{c}_i$, for the various sources along with the source flow. These values were used to calculate the observed "cost" of undetected violations and observed number of violators. Table 9.11 compares the sampling frequencies obtained by the Allocation by Flow method and the Resource Allocation Program (two lower bounds on sampling frequency were chosen for the Resource Allocation Program: zero and one.) as well as comparing the performance criteria. The budget was assumed to be $15,000. From Table 9.11 it is seen that the Resource Allocation Program produces a better allocation for this example than the Allocation by flow method. The improvement is greater for the observed "cost" of undetected violations than for the observed number of violators.

It is recommended that more comparison studies be done in the future using larger data bases. This study was hampered by the fact that only one month of data was used. Since samples are highly correlated, day-to-day, for many industries, a small number of independent samples went into the calculation of the observed damage and the observed frequency of violation. (Note that over half the sources were either always in violation or never in violation.) It therefore is expected that much better performance of the Resource Allocation Program would have been shown if more months of data were used in the comparison.

Table 9.10 OBSERVED FREQUENCY OF VIOLATION AND AVERAGE DAMAGE

Source	Source flow, Ml/day	Observed frequency of violation, %	Observed average damage
3	0.075	0.0	0.00
12	4.92	41.4	5.73
16	0.725	22.2	3.01
18	35.55	50.0	3.90
19	0.133	0.0	1.32
22	40.75	100.0	6.85
23	0.425	13.3	4.01
24	3.04	0.0	0.98
25	165.0	100.0	3.70
26	7.15	100.0	0.43
27	5.57	100.0	2.61
28	110.9	100.0	4.15
29	4.11	5.0	5.79
30	35.0	87.1	1.13

Table 9.11 PERFORMANCE COMPARISON

Source	Allocation By	Optimal Allocation $s_i \geq 1$	Optimal Allocation $s_i \geq 0$
		Sampling Frequencies	
3	1	1	0
12	1	2	2
16	1	1	0
18	2	1	2
19	1	1	0
22	2	2	3
23	1	2	3
24	1	1	0
25	2	1	2
26	2	1	1
27	2	1	1
28	2	3	3
29	1	3	3
30	2	1	1
Observed "cost" of undetected violations	19.00	17.97	17.23
Observed number of violators caught	7.55	7.64	7.77

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SECTION X
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GLOSSARY

TERMINOLOGY

<u>BOD</u> - Biochemical oxygen demand.

<u>COD</u> - Chemical oxygen demand.

<u>DO</u> - Dissolved oxygen.

K_{BOD-DO} - BOD-dissolved oxygen transfer coefficient

<u>Damage</u> - A measure of effect of pollutants on water quality.

<u>Effluent Standard</u> - A restriction on the quantities or concentrations of constituents from an effluent source.

<u>Monitor</u> - The government agency having responsibility for enforcing laws realting to the abatement of pollution.

<u>Permit</u> - A document or requirement regulating the discharge of pollutants.

<u>Resources</u> - Money required to obtain and process effluent samples obtained during compliance monitoring.

<u>Resource Allocation Program</u> - Name given to procedure for setting compliance monitoring priorities.

<u>Source</u> - A discharger or possible discharger of pollutants subject to effluent standards.

<u>Water Quality Limited Segment</u> - A segment of a river where it is known that water quality does not meet applicable water quality standards and which is not expected to meet water quality standards even after the application of the effluent limitations required by the Water Pollution Control Act.

MATHEMATICAL NOTATION

A - Maximum allowed cost of undetected violations.

α - Level of significance of a statistical hypothesis test.

B - Monitoring agency's budget.

C – Total "cost" of undetected violations.

$C_i(S_i)$ – "Cost" of undetected violations for source i.

c_i – Expected damage from all the constitutents of source i.

CO_{ij} – Stream concentration at discharge point, constituent j, source i.

CU_{ij} – Upstream concentration, constituent j, source i.

CX_{ij} – Downstream concentration, constituent j, source i.

D_{ij} – Expected damage due to constituent j, source i.

$D_{ij\ell}$ – Expected damage due to constituent j, from source i into stream ℓ.

$d_j(\cdot)$ – Damage function for constituent j.

$D_{i,BOD}$ – Dissolved oxygen deficit due to BOD, source i.

$D_{i,COD}$ – Dissolved oxygen deficit due to COD, source i.

$DOMIN_{i,BOD}$ – Minimum DO level downstream from source i.

$\dot{\phi}_{ij}(\cdot)$ – Density function of mass loading M_{ij}.

γ – Parameter denoting relative weight given compliance data over self-monitoring data.

h_n – Factor relating confidence in mean to number of measurements.

h_ν – Factor relating confidence in variance to the number of measurements.

i – Index denoting source.

j – Index denoting constituent.

k – Index denoting outfall.

ℓ – Index denoting receiving water.

k_n – Constant for determining the confidence in mean.

k_ν – Constant for determining the confidence in variance.

ℓ_i – Lower bound on sampling frequency for source i.

L_i – Upper bound on the sampling frequency for source i.

M_{ij} - Effluent mass loading, constituent j. source i.

mg/l - Milligrams per liter.

Ml - Megaliters.

m - Estimate of mean

μ - Mean of a random process.

$\hat{\mu}$ - Estimate of mean, μ.

$\mu_i(\cdot)$ - Marginal return function.

N - Number of sampling days in monitoring period.

n - Number of measurements or confidence in mean estimate.

n_s - Number of sources.

$p(\cdot)$ - Probability event occurs.

P_i - Probability of no violation for all constituents of source i.

P_{ij} - Probability of no violation for constituent j, source i.

P_{ijk} - Probability of no violation due to constituent j, outfall k, source i.

QS_i - Effluent flow rate, source i.

QU_i - Upstream flow rate, source i.

QX_i - Downstream flow rate, source i.

$R(\cdot)$ - Resources required to monitor all the sources.

r_i - Resources required to monitor source i once.

s_i - Number of times the ith source is sampled in monitoring period.

σ - Standard deviation of a random process.

$\hat{\sigma}$ - Estimate of standard deviation, σ.

τ_{ij} - Standard for constituent j, source i.

v - Estimate of variance.

V_i - Event ith source is in violation.

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$\overline{V}_i$ - Event ith source is not in violation.

ν - Confidence in variance estimate.

x - Distance downstream from source or a random process.

y - Maximum of a set of data.

z_i - Compliance monitoring data.

APPENDIX A

ESTIMATION OF DISTRIBUTION PARAMETERS

In this Appendix the estimation of the parameters of the normal and log-normal probability density functions is discussed for the case where the available data consist of the sample mean and maximum of a set of observations. These two problems are treated in Sections A.1 and A.2. Section A.3 deals with the examination of the parameters when the available data consist of the maximum and the minimum value of a set of observations.

A.1   THE NORMAL CASE

In this case the process x is assumed normally distributed with mean μ and variance σ^2. The available data to estimate μ and σ is

$$z = [m, \xi] \qquad \text{(A.1.1)}$$

where m is the sample mean and

$$\xi = \max \{x_1, \ldots, x_n\} . \qquad \text{(A.1.2)}$$

Approximate maximum likelihood estimates of μ and σ^2 will now be obtained.

The calculation of the likelihood function

$$p(m, \xi | \mu, \sigma^2) \qquad \text{(A.1.3)}$$

requires the joint probability density function for m and ξ. This density is not obtainable in closed form. Approximate maximum likelihood estimates can be obtained by estimating μ by m, the sample mean. $\hat{\sigma}$, the estimate of σ, is then that value of σ that maximizes

$$p(\xi | \mu = m, \sigma^2) \qquad \text{(A.1.4)}$$

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The above density is obtained as follows:

$$\text{Prob } \{\max(x_1, \ldots, x_n) \leq \xi | \mu, \sigma\} = F^n \left(\frac{\xi - \mu}{\sigma} \right) \qquad \text{(A.1.5)}$$

where

$$F(\alpha) \stackrel{\Delta}{=} \int_{-\infty}^{\alpha} (2\pi)^{-1/2} \, e^{-x^2/2} \, dx \qquad \text{(A.1.6)}$$

Therefore,

$$p(\xi | \mu, \sigma) = \frac{d}{d\xi} F^n \left(\frac{\xi - \mu}{\sigma} \right) \qquad \text{(A.1.7)}$$

For convenience denote

$$x_i' \stackrel{\Delta}{=} x_i - \mu \qquad \text{(A.1.8)}$$

then

$$\xi' = \max \left| x_1', \ldots, x_n' \right| = \xi - \mu \qquad \text{(A.1.9)}$$

and, hence

$$p(\xi' | \sigma) = \frac{\partial}{\partial \xi'} F^n \left(\frac{\xi'}{\sigma} \right) \qquad \text{(A.1.10)}$$

Let

$$f(y) = \frac{d}{dy} F(y) = \frac{1}{\sqrt{2\pi}} \, e^{-y^2/2} \qquad \text{(A.1.11)}$$

Denoting

$$\zeta \stackrel{\Delta}{=} \frac{\xi'}{\sigma} = \frac{\xi - \mu}{\sigma} \qquad \text{(A.1.12)}$$

(A.1.10) becomes

$$p(\xi' | \sigma) = n F^{n-1}(\zeta) \, f(\zeta) \, \frac{1}{\sigma} \qquad \text{(A.1.13)}$$

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$$= nF^{n-1}(\zeta) \quad f(\zeta) \frac{\zeta}{\xi}$$

The likelihood equation is therefore

$$\frac{\partial}{\partial \sigma} p(\xi'|\sigma) = \frac{\partial p}{\partial \zeta} \frac{\partial \zeta}{\partial \sigma} - \frac{\partial p}{\partial \zeta} \frac{\xi'}{\sigma^2} = 0 \qquad (A.1.14)$$

which is equivalent to

$$\frac{\partial p}{\partial \zeta} = 0 \qquad (A.1.15)$$

Note that

$$\frac{df(\zeta)}{d\zeta} = \zeta f(\zeta) \qquad (A.1.16)$$

Thus (A.1.15) can be written as follows

$$(n-1)F^{n-2}(\zeta) \quad f^2(\zeta)\zeta - F^{n-1}(\zeta)f(\zeta)\zeta^2 + F^{n-1}(\zeta)f(\zeta) = 0$$

$$(A.1.17)$$

or

$$\frac{(\zeta^2-1)F(\zeta)}{\zeta f(\zeta)} = n-1 \qquad (A.1.18)$$

The left hand side of (A.1.18) is plotted in Figure A.1.1. Using this figure, it is easy to determine $\hat{\sigma}$, the estimate of σ, given ξ, μ and n. This is done by obtaining, $\hat{\zeta}$ for the given n from Figure A.1.1, then

$$\hat{\sigma} = \frac{\xi-\mu}{\zeta} \qquad (A.1.19)$$

For example, suppose n = 31, μ = 5 and ξ = 10. From the figure, (n-1) = 30 implies $\zeta \approx 2.035$. Thus

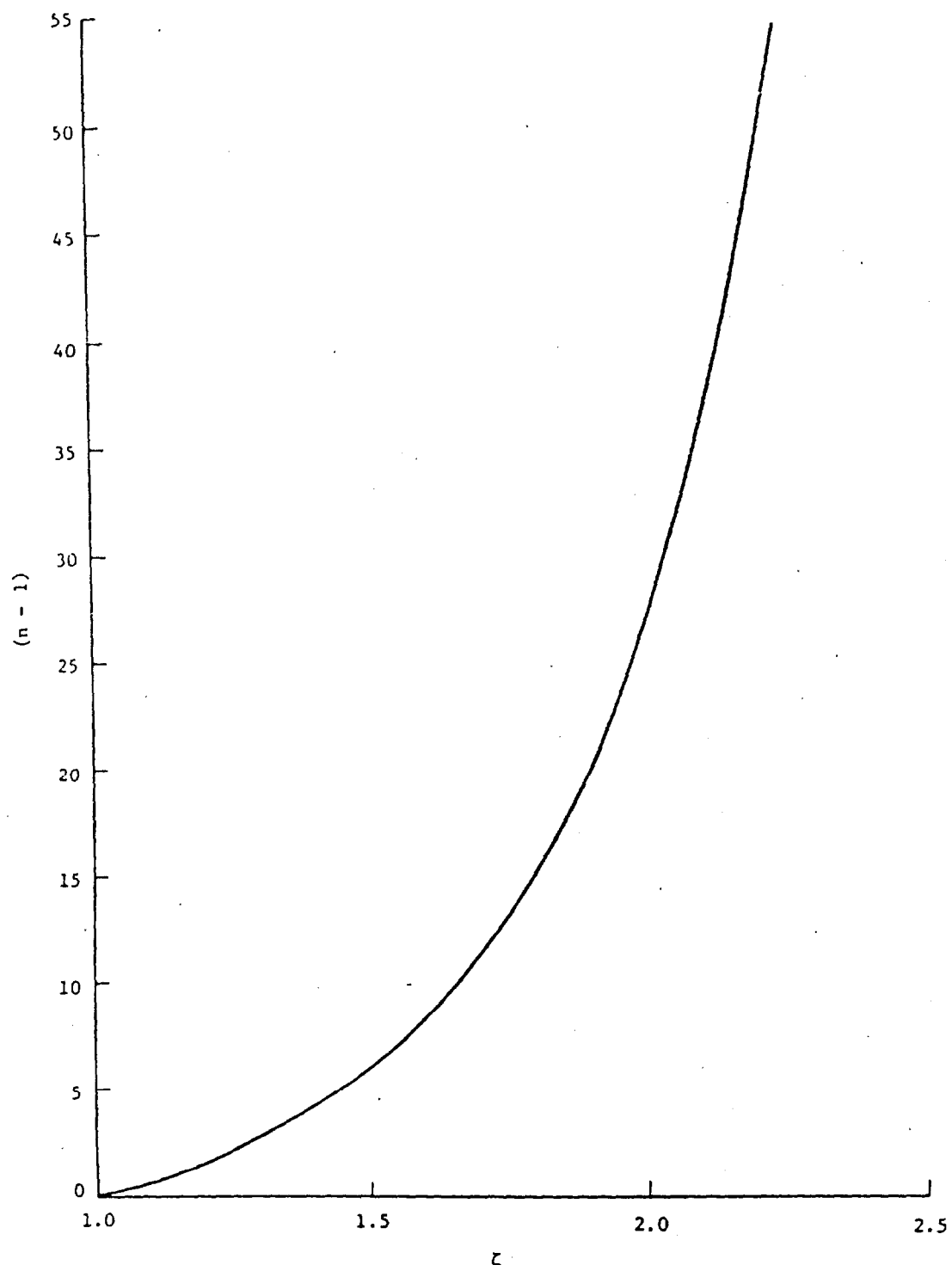$$\hat{\sigma} = \frac{\xi-\mu}{2.035} = \frac{5}{2.035} \approx 2.46$$

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Figure A.1.1 Plot of equation (A.1.18).

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A.2   THE LOGNORMAL CASE

In the lognormal case*, if x_i are the measurements, then

$$y_i = \ln x_i \sim \mathcal{N}(\mu, \sigma^2) \tag{A.2.1}$$

and $\Theta = [\mu, \sigma]$ is the unknown parameter. Note that μ and σ are the mean and standard deviation of the logs of the measurements rather than of the measurements as in the normal case. Assume that the statistic is, as before

$$z = [m, \xi] \tag{A.2.2}$$

i.e., the sample mean of the measurements, m, and the largest measurement

$$\xi = \max\{x_1, \ldots, x_n\} \tag{A.2.3}$$

The estimate of the mean of x_i is taken to be the sample mean m, therefore

$$m = E\{x_i\} = \exp\left\{ \hat{\mu} + \frac{\sigma^2}{2} \right\} \tag{A.2.4}$$

or

$$\hat{\mu} + \frac{\sigma^2}{2} = \ln m \tag{A.2.5}$$

The maximum likelihood estimate of σ is obtained by maximizing

$$p\left(\xi \,\middle|\, \mu = \left(\ln m - \frac{\sigma^2}{2} \right),\ \sigma \right) \tag{A.2.6}$$

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*Natural logarithms are used throughout the derivation. The final results are given in terms of common logrithms.

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with respect to σ. First, the distribution of ξ is

$$P\{\max(x_1,\ldots,x_n) \leq \xi\} = P\{\max(y_1,\ldots,y_n) \leq \ln \xi\} \qquad (A.2.7)$$

$$= F^n \left(\frac{\ln \xi - \mu}{\sigma} \right)$$

where F is the standard Gaussian distribution (A.1.6). Therefore, denoting

$$\rho \overset{\Delta}{=} \frac{\xi}{m} \qquad (A.2.8a)$$

$$\eta \overset{\Delta}{=} \frac{\ln \xi - \mu}{\sigma} = \frac{\ln \xi - \ln m + (\sigma^2/2)}{\sigma} \qquad (A.2.8b)$$

$$= \frac{\ln \rho + (\sigma^2/2)}{\sigma}$$

the density of ξ is

$$p(\xi|\mu,\sigma) = \frac{d}{d\xi} F^n(\eta)$$

$$= n \, F^{n-1}(\eta) \, f(\eta)\frac{d\eta}{d\xi} \qquad (A.2.9)$$

From (A.2.8b) one has

$$\frac{d\eta}{d\xi} = \frac{1}{\sigma\xi} \qquad (A.2.10)$$

Combining (A.2.9) and (A.2.10) yields

$$p(\xi|\mu,\sigma) = n \, F^{n-1}(\eta) \, f(\eta) \, (\sigma\xi)^{-1} \qquad (A.2.11)$$

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The likelihood equation is, therefore

$$\frac{d}{d\sigma} P(\xi|\mu,\sigma) = \left[n(n-1)F^{n-2}(\eta) f^2(\eta) (\sigma\xi)^{-1} \right.$$

$$\left. - n F^{n-1}(\eta)\eta f(\eta)(\sigma\xi)^{-1} \right] \frac{d\eta}{d\sigma} \qquad (A.2.12)$$

$$- n F^{n-1}(\eta) f(\eta) \sigma^{-2} \xi^{-1} = 0$$

where use has been made of (A.1.16). Also

$$\frac{d\eta}{d\sigma} = \frac{\sigma^2 - [\ln \rho + (\sigma^2/2)]}{\sigma^2} = 1 - \frac{\eta}{\sigma} \qquad (A.2.13)$$

Inserting (A.2.13) into (A.2.12) yields the following equation for σ

$$[(n-1) f(\eta) - F(\eta)\eta] (\sigma - \eta) - F(\eta) = 0 \qquad (A.2.14)$$

where $\eta = \eta(\sigma)$ according to (A.2.8b).

The solution $\hat{\sigma}$ of (A.2.14) for common logarithms is presented graphically in Figure A.2.1 as a function of the number of measurements n and the ratio ρ between the maximum and the mean. For example, assume m = 10, ξ = 30 and n = 30. Then ρ =3 and $\hat{\sigma}$ = 0.27. The estimate $\hat{\mu}$ is obtained using (A.2.5):

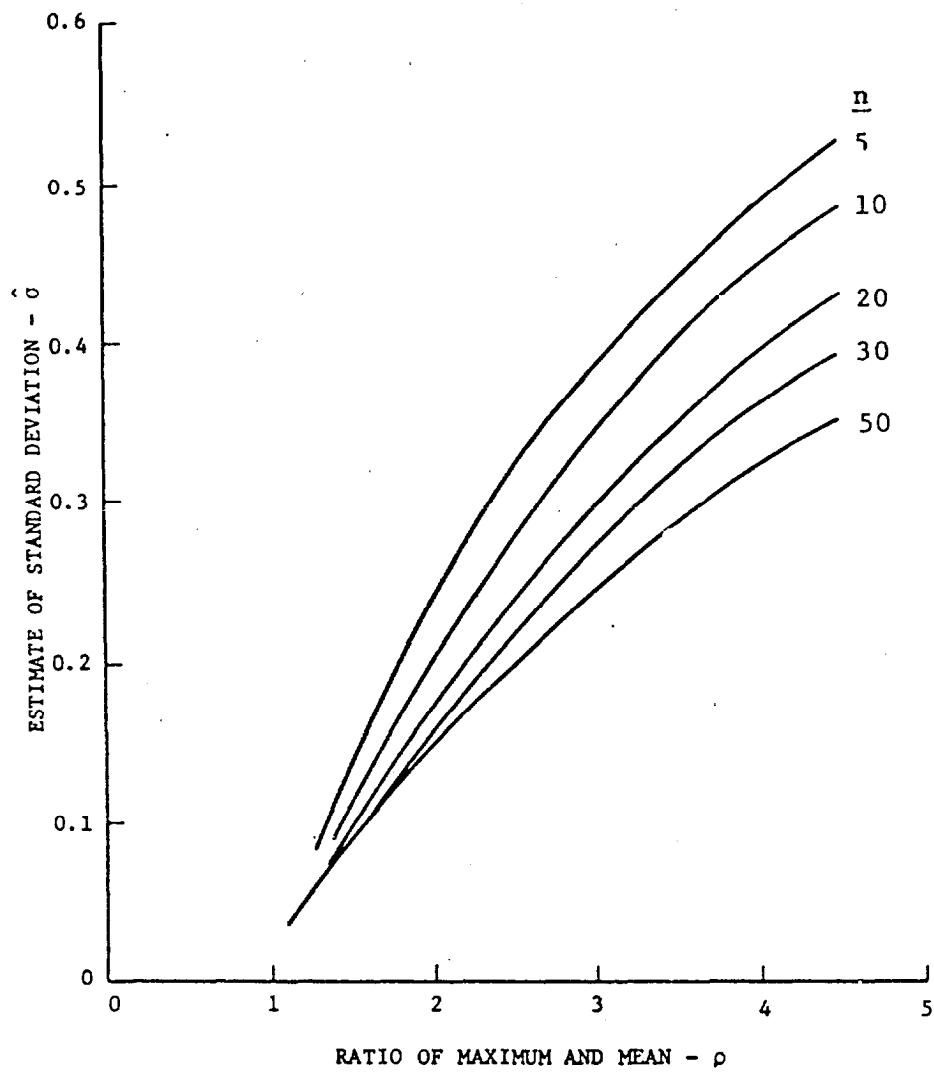$$\hat{\mu} = \log m - \ln 10 \frac{\hat{\sigma}^2}{2} = 1 - (2.3)(0.0365) = 0.916$$

Figure A.2.1   Maximum likelihood estimate of standard deviation from mean and maximum in lognormal case.

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A.3 ESTIMATE OF MEAN AND STANDARD DEVIATION FROM MAXIMUM AND MINIMUM

Let $x_1, \ldots, x_N$ be independent samples from a **normal** $\mathcal{N}(\mu, \sigma^2)$ distribution and let $y_1 = \min(x_1, \ldots, x_n)$ and $y_N = \max(x_1, \ldots, x_N)$. Then simple estimates of μ and σ can be obtained from the midrange $m = (y_1 + y_N)/2$ and the range $R = y_N - y_1$.

Estimate of Mean

The obvious estimate of the mean is the midrange. Kendall and Stuart [A1] gives the relative efficiency of this estimate as compared to the efficiency of the sample mean for several values of N (see Table A.3.1).

Table A.3.1 RELATIVE EFFICIENCY OF MIDRANGE
AS AN ESTIMATE OF μ

N	Relative efficiency	N	Relative efficiency
2	1.000	10	.734
4	.915	20	.591
6	.840	∞	0

Estimate of Standard Deviation

The estimate of the standard deviation from y_1 and y_N has historically [A2], [A3] been in the form

$$\hat{\sigma} = R/C_N \qquad (A.3.1)$$

where R is the range and $C_N = E(\tilde{R})$ where $\tilde{R}$ is the range of N samples for a $\mathcal{N}(0,1)$ distribution. $\hat{\sigma}$ is therefore an unbiased estimate of σ. A table of C_N versus N is given in Table A.3..2 [A3].

Table A.3.2. C_N VERSUS N

N	C_N	N	C_N
0	-	21	3.778
1	-	22	3.819
2	1.128	23	3.858
3	1.693	24	3.895
4	2.059	25	3.930
5	2.326	26	3.964
6	2.534	27	3.997
7	2.704	28	4.027
8	2.847	29	4.057
9	2.970	30	4.086
10	3.078	31	4.113
11	3.173	32	4.139
12	3.258	33	4.165
13	3.336	34	4.189
14	3.407	35	4.213
15	3.472	36	4.236
16	3.532	37	4.259
17	3.588	38	4.280
18	3.640	39	4.301
19	3.689	40	4.322
20	3.735		

In [A4], the relative efficiency of this estimate is given as compared to the efficiency of the sample standard deviation.   Several values are shown in Table A.3.3.

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TABLE A.3.3 RELATIVE EFFICIENCY
OF THE ESTIMATE σ

N	Relative efficiency	N	Relative efficiency
2	1.000	10	0.850
4	0.975	20	0.700
6	0.933	50	0.490

REFERENCES

A1. Kendall, M. and Stuart, A., The Advanced Theory of Statistics, Volume 2, Hafner Publishing Company, New York, 1967.

A2. Mosteller, G., "On Some Useful 'Inefficient' Statistics," Ann. Math. Stat., Vol. 17, 1946, pp. 377-408.

A3. Tippett, L. H. C., "On the Extreme Individuals and the Range of Samples Taken from a Normal Distribution," Biometrika, Vol. 17, 1925, pp. 364-387.

A4. Snedecor, G. W. and Cochran, W. G., Statistical Analysis, University of Iowa Press, 1972.

APPENDIX B

INVESTIGATION OF THE CORRELATION
BETWEEN EFFLUENT CONSTITUENTS

In this appendix a procedure is presented to test for the uncorrelatedness of normal random variables with unknown mean and unknown variance. Subsequently, it is applied to data from the Palo Alto Municipal Waste Treatment Plant.

B.1 THE UNCORRELATEDNESS TEST

Consider two normal random variables x, and **y,** from which n independent samples x_i, i = 1, ..., n and y_i, i = 1, ..., n are available. The true means and variances are unknown and can be estimated by the well-known equations

$$\bar{x} = \frac{1}{n} \sum x_i, \qquad \bar{y} = \frac{1}{n} \sum y_i$$

$$s_x^2 = \frac{1}{n} \sum (x_i - \bar{x})^2, \qquad s_y^2 = \frac{1}{n} \sum (y_i - \bar{y})^2$$

(B.1.1)

We want to test whether their correlation

$$\rho \triangleq \frac{E\left[(x - Ex)(y - Ey)\right]}{\left[E(x - Ex)^2 E(y - Ey)^2\right]^{1/2}}$$

(B.1.2)

is zero or not, i.e.,

$$H_0 : \rho = 0$$

(B.1.3)

vs.

$$H_1 : \rho \neq 0$$

(B.1.4)

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Define the sample correlation as

$$r \triangleq \frac{1}{ns_x s_y} \sum (x_i - \bar{x})(y_i - \bar{y}) \qquad (B.1.5)$$

It has been pointed out in Kendall and Stuart [B1] that the distribution of this sample correlation converges very slowly to the normal and thus a test based on the normality assumption is not accurate. The exact test is presented next. As shown in [B1]

$$t = \left[\frac{(n-2)r^2}{1-r^2} \right] \qquad (B.1.6)$$

has a t-distribution with $\nu = n-2$ degrees of freedom. Thus the above simple transformation enables one to test H_o against H_1 using readily available tables.

To illustrate the procedure, consider for example $n = 30$. The t values corresponding to various values of the sample correlation r are presented in Table B.1.1. Also, the significance levels above which H_o would be rejected (and H_1 accepted) for these values of r are given.

Table B.1.1 UNCORRELATEDNESS TEST FOR N=30 SAMPLES

r	t	α%
0.5	3.06	<1
0.4	2.31	3
0.35	1.99	6
0.3	1.66	11
0.25	1.37	18

If the observed value is r = 0.35, then at 5% level of significance (probability of error of type I) H_o would be accepted.

B.2   EXAMPLE OF UNCORRELATEDNESS TESTS FOR EFFLUENT CONSTITUENTS

Tests were run on a number of constituents from the Palo Alto Municipal Waste Treatment Plant. The data consisted of daily composite samples of the following

1.   Flow
2.   Suspended Solids
3.   BOD (Biological Oxygen Demand)
4.   TOC (Total Organic Carbon)
5.   COD (Chemical Oxygen Demand)

Data was obtained from a dry month (July 1973) and a wet month (November 1973) each with 30 samples. The correlation coefficients were computed for the actual measurements, under the normal assumption and for the logarithms of the measurements, under the lognormal assumption. (The goodness of these assumptions was examined in Section V.1).

The resulting correlation coefficients are presented in Tables B.2.1 and B.2.2. An examination of these tables reveals that the sample correlations are such that only at relatively low significance levels ($\alpha \sim 1\%$) would the hypothesis of uncorrelatedness be accepted in some cases. This can be seen from the uncorrelatedness test illustrated in Table B.2.1. However, the variation of the correlation coefficients seems to be large from season to season and no clear pattern seems to emerge. For example, the r_{23} term (SS vs. BOD) is positive in a dry month while in a wet month it can become negative. Also notice that there is no appreciable difference in the correlation tests when done under normal or lognormal assumption. The hypothesis that the effluent constituents are highly (near unity) correlated is even less likely than their being uncorrelated.

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Table B.2.1   SAMPLE CORRELATIONS OF THE MEASUREMENTS

Month	Variable sampled	2	3	4	5
Dry	1	0.28	0.33	0.55	0.58
	2		0.46	0.39	0.62
	3			0.43	0.50
	4				0.47
Wet	1	-0.24	0.45	0.29	0.22
	2		-0.19	0.27	0.25
	3			0.35	0.13
	4				0.51

Table B.2.2   SAMPLE CORRELATIONS OF LOGS OF THE MEASUREMENTS

Month	Variable sampled	2	3	4	5
Dry	1	0.30	0.32	0.58	0.60
	2		0.45	0.40	0.68
	3			0.51	0.45
	4				0.49
Wet	1	-0.24	0.50	0.28	0.23
	2		-0.18	0.37	0.33
	3			0.30	0.10
	4				0.59

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REFERENCE

B1.   Kendall, M. and Stuart, A., <u>The Advanced Theory of Statistics</u>, Volume 2, Hafner Publishing Company, New York, 1967.

APPENDIX C

EXPECTED DAMAGE AND PROBABILITY OF VIOLATION CALCULATIONS

C.1   INTRODUCTION

The sampling frequencies to choose, in determining whom to monitor, mini-
mize the "cost" of undetected violations. This "cost" was derived in
Section VI to be:

$$C = \sum_{\substack{sources \\ i}} c_i \, p_i^{s_i} \qquad (C.1.1)$$

where c_i is the expected damage caused by the ith source, p_i is the
probability that the ith source will <u>not</u> violate any effluent standard,
and s_i is the number of times the ith source is to be monitored c_i
equals the maximum of the expected damages due to the various constituents
of the ith source, or

$$c_i = \max_j D_{ij} \qquad (C.1.2)$$

where D_{ij} is the expected damage due to the jth constituent of the ith
source. p_i, assuming independence between the various constituents, is

$$p_i = \prod_j p_{ij} \qquad (C.1.3)$$

where p_{ij} is the probability the standard on the jth constituent is not
violated. If the constituents are completely correlated, then

$$P_i = \min_{j} P_{ij} \qquad (C.1.4)$$

This appendix describes in detail both the calculation of the expected damage, D_{ij}, due to constituent j from source i and P_{ij}, the probability that constituent j, source i, does not violate its standard. It is organized as follows: Section C.2 calculates D_{ij} and P_{ij} under the assumption that only one set of effluent standards is given for the source. This corresponds to the case where there is only one outfall or the permits are written for the combined discharge from several outfalls. Section C.3 describes how these calculations are generalized to the case when standards are set for many outfalls from a single industry or municipal treatment plant. Section C.4 evaluates certain integrals that arise often in the expected damage and probability of violation calculation.

C.2 EXPECTED DAMAGE AND PROBABILITY OF VIOLATION DERIVATION: ONE SET OF STANDARDS

This section describes the derivation of the expected damage from a source and the probability of violation when there is either a single outfall from the source or there are several outfalls, all to the same river, and there is one set of standards for the total discharge from the source. When there are several outfalls but only one set of standards for the total effluent, the monthly self-monitoring reports are on the total effluent, and so the several outfalls can be treated as one.

The section is divided into four subsections. The first subsection considers the majority of constituents. All the calculations needed to determine the expected damage and probability of violation for this set of constituents are the same, pH, BOD, and temperature require slightly different calculations, and they will be treated separately in the remaining subsections.

C.2.1 Noncoupled Constituents

This subsection derives expected damage and probability of violation for all the indicators listed in Table 6.1 except pH, temperature, and dissolved oxygen.

Inputs

The data needed to calculate expected damage and probability of violation are:

For source i:

$$P_i = \text{index set of pollutants}$$

$$\mu_{ij} = \text{mean of mass loading of } j^{th} \text{ pollutant (kg)}$$

$$\sigma_{ij} = \text{standard deviation of mass loading of } j^{th} \text{ pollutant (kg)}$$

$$\gamma_{ij} = \text{distribution of } j^{th} \text{ pollutant--normal or lognormal}$$

$$QU_i = \text{flow of stream above source (Ml/day)}$$

$$QS_i = \text{effluent flow (Ml/day)}$$

$$CU_{ij} = \text{concentration of the pollutant upstream from source (mg/l)}$$

$$EFST_{ij} = \text{effluent standard for } j^{th} \text{ pollutant (kg)}$$

For each pollutant j:

$$d_j(k) = \text{concentration of pollutant when damage equals } 2(k-1),$$
$$k = 1, 2, ..., 6.$$

$d_j(k)$ is the value of the abscissa of the damage function at the kth breakpoint. The damage function breakpoints for the constituents of interest were given in Table 6.1. The damage function of the jth pollutant is then

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$$D_j(\alpha) = \sum_{k=1}^{5} \left\{ \frac{2(\alpha - d_j(k))}{(d_j(k+1) - d_j(k))} + 2(k-1) \right\} \phi(d(k), d(k+1), \alpha)$$

$$+ \ 10 \ \phi(d(6), \infty, \alpha) \tag{C.2.1}$$

where α is the concentration of pollutant and ϕ is the characteristic function:

$$\phi(x, y, \alpha) = \begin{cases} 1 & ; \quad x \le \alpha < y \\ \\ 0 & ; \quad \text{otherwise.} \end{cases} \tag{C.2.2}$$

Maximum Downstream Concentration

The maximum downstream concentration for the jth pollutant – i^{th} source is given by the conservation law:

$$CO_{ij} = \frac{M_{ij} + CU_{ij} \ QU_i}{QU_i + QS_i} \tag{C.2.3}$$

where M_{ij} is the mass loading of the jth pollutant – i^{th} source (M_{ij} is a random variable with mean μ_{ij}, standard deviation σ_{ij} and distributional form γ_{ij}) and CU_{ij} is the concentration of the jth pollutant upstream from the ith source. (C.2.3) can be rewritten to yield

$$CO_{ij} = a_i M_{ij} + b_{ij} \tag{C.2.4a}$$

where

$$a_i = 1/(QU_i + QS_i) \tag{C.2.4b}$$

and

$$b_{ij} = CU_{ij} \left(\frac{QU_i}{QU_i + QS_i} \right) \qquad\qquad \text{(C.2.4c)}$$

Expected Damage

The expected damage due to pollutant j from the ith source is then

$$D_{ij} = E(D_j(CO_{ij}))$$

$$= \int D_j(CO_{ij}) \; \phi_{ij}(M) \; dM \qquad\qquad \text{(C.2.5)}$$

where $E(\cdot)$ is the expectation operator and ϕ_{ij} is the probability density function of the mass loading M_{ij}. Using (C.2.4),

$$D_{ij} = \int D_j(a_i M + b_{ij}) \; \phi_{ij}(M) \; dM \qquad\qquad \text{(C.2.6)}$$

Combining (C.2.1) and (C.2.6),

$$D_{ij} = \sum_{k=1}^{6} \int_{\alpha_{ijk}}^{\beta_{ijk}} \left\{ e_{ijk}M + f_{ijk} \right\} \; \phi_{ij}(M) \; dM \qquad\qquad \text{(C.2.7a)}$$

where

$$\alpha_{ijk} = \frac{d_j(k) - b_{ij}}{a_i} \qquad\qquad k = 1, 2, \ldots, 6 \qquad\qquad \text{(C.2.7b)}$$

$$\beta_{ijk} = \begin{cases} \dfrac{d_j(k+1) - b_{ij}}{a_i} & ; \quad k = 1, 2, \ldots, 5 \\[2mm] \infty & ; \quad k = 6 \end{cases} \qquad\qquad \text{(C.2.7c)}$$

$$e_{ijk} = \begin{cases} 2a_i/(d_j(k+1) - d_j(k)) & ; \quad k = 1,2,\dots,5 \\ \\ 0 & ; \quad k = 6 \end{cases} \qquad \text{(C.2.7d)}$$

$$f_{ijk} = \begin{cases} \dfrac{2(b_{ij} - d_j(k))}{(d_j(k+1) - d_j(k))} + 2(k-1) & ; \quad k = 1,\dots,5 \\ \\ 10 & ; \quad k = 6 \end{cases} \qquad \text{(C.2.7e)}$$

If α_{ijk} or β_{ijk} are less than 0, set them to 0. (C.2.7) can be re-written

$$D_{ij} = \sum_{k=1}^{6} I_{\gamma_{ij}}(e_{ijk}, f_{ijk}, \alpha_{ijk}, \beta_{ijk}, \mu_{ij}, \sigma_{ij}) \qquad \text{(C.2.8)}$$

where

$$I_\gamma(e, f, \alpha, \beta, \mu, \sigma) = \int_\alpha^\beta (ex + f)\, \varphi_\gamma(x)\, dx \qquad \text{(C.2.9)}$$

and where ϕ_γ is the normal density function with mean μ and variance σ^2 if $\gamma =$ Normal, and is lognormal, with mean and variance of corresponding normal distribution being μ and σ^2, respectively, if $\gamma =$ Lognormal. (C.2.9) is evaluated in Section C.4 for the normal and lognormal cases.

Probability of a Violation

The probability that a standard for the jth pollutant i^{th} source is not violated is

$$P_{ij} = \int_{-\infty}^{EFST_{ij}} \phi_{ij}(M)\, dM$$

$$= I_{\gamma_{ij}}(0, 1, -\infty, EFST_{ij}, \mu_{ij}, \sigma_{ij}) \qquad \text{(C.2.10)}$$

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where I_γ is defined in (C.2.9)

C.2.2   5-Day Biochemical Oxygen Demand - BOD_5

The presence of BOD_5 in the water causes a depletion in the dissolved oxygen (DO). The difference between the saturated level of dissolved oxygen, DOSAT, in the water and the actual level is called the dissolved oxygen deficit or DO-deficit. The degree of depletion caused by a given amount of BOD_5 from a source depends on several stream parameters such as stream depth, flow rate, temperature, and the distance downstream from the source. The relationship between BOD_5 and DO-deficit can be expressed (see Section VI.1) in the form

$$D_{max} = K_{BOD-DO} CO \qquad (C.2.11)$$

where CO is the concentration o f BOD_5 at the source, D_{max} is the maximum DO deficit downstream from the source, and K_{BOD-DO} is the BOD_5-DO transfer coefficient.

Inputs

The data needed to calculate the expected damage and probability of violation due to BOD_5 is:

For source i:

$\mu_{i,BOD}$ = mean of mass loading of BOD_5 (kg)

$\sigma_{i,BOD}$ = standard deviation of mass loading of BOD_5 (kg)

$\gamma_{i,BOD}$ = distribution of mass loading of BOD_5

$CS_{i,DO}$ = mean of DO concentration of the source (mg/l)

$\bar{K}_{BOD-DO_i}$ = BOD_5-DO transfer coefficient

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QU_i = flow of stream above source (Ml/day)

QS_i = effluent flow (Ml/day)

$DOSAT_i$ = saturation level of DO in the stream (mg/l)

$EFST_{i,BOD}$ = effluent standard for BOD_5 (kg)

$CU_{i,BOD}$ = upstream concentration of BOD_5 (mg/l).

Maximum Downstream Concentration

The concentration of BOD at the point where the outfall empties into the stream is given by

$$CO_{i,BOD} = \frac{M_{i,BOD} + CU_{i,BOD} \ QU_i}{QU_i + QS_i} \qquad (C.2.12)$$

The concentration of DO is similarly

$$CO_{i,DO} = \frac{CS_{i,DO} \ QS_i + CU_{i,DO} \ QU_i}{QU_i + QS_i} \qquad (C.2.13)$$

$CU_{i,BOD}$ and $CU_{i,DO}$ are unknown and must be chosen in a way that is consistent with determining the sampling frequencies. It was suggested in Section VI that they be chosen so that a given level of damage will occur when the loading $M_{i,BOD}$ is zero and the concentration of DO in the source, $CS_{i,DO}$, is equal to $DOSAT_i$. Since there are two unknowns and only one requirement, the upstream DO concentration, $CU_{i,DO}$, shall be arbitrarily set equal to $DOSAT_i$. The value of $CU_{i,BOD}$ will be set to give the desired downstream DO concentration under zero load.

The minimum concentration of DO downstream from the source can be approximated by (see Section VI.1):

$$CO_{i,DOMIN} = CO_{i,DO} - K_{BOD-DO_i} \ CO_{i,BOD} \qquad (C.2.14)$$

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Using (C.2.12) and (C.2.13) and noting that $CU_{i,DO} = DOSAT_i$ we obtain

$$CO_{i,DOMIN}$$

$$= \frac{CS_{i,DO}\, QS_i + (DOSAT_i - K_{BOD-DO_i}\, CU_{i,BOD})\, QU_i - K_{BOD-DO_i}\, M_{i,BOD}}{QU_i + QS_i}$$

$$\text{(C.2.15)}$$

or

$$CO_{i,DOMIN} = a_{i,BOD}\, M_{i,BOD} + b_{i,BOD} \qquad\qquad \text{(C.2.16a)}$$

where

$$a_{i,BOD} = - K_{BOD-DO_i}/(QU_i + QS_i) \qquad\qquad \text{(C.2.16b)}$$

and

$$b_{i,BOD} = \frac{1}{(QU_i + QS_i)}\left[CS_{i,DO}QS_i + \left(DOSAT_i - K_{BOD,DO_i}\, CU_{i,BOD} \right) QU_i \right]$$

$$\text{(C.2.16c)}$$

Probability of Violation

The probability that the BOD_5 effluent standard will not be violated is given by (C.2.10) with j = BOD.

C.2.3 pH

pH is a measure of the acidity (alkalinity) of a solution. It is defined as the negative of the log of the concentration[†] of H^+ ions. pOH is defined to be the negative of the log of the concentration of OH^- ions. pOH and pH are related by the equation

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[†] The concentration is in moles/liter.

$$pOH + pH = 14 \qquad\qquad (C.2.17)$$

For pure water (H_2O), $pOH = pH = 7$. $pH < 7$ implies an acidic solution and $pH > 7$ implies a basic or alkaline solution. If two acidic solutions are combined, then the number of H^+ ions is equal to the sum of the H^+ ions from the two original solutions.* Similarly, if two basic solutions are combined, the number of OH^- ions add. Therefore, if, for example, we combine X liters of an acid with $pH = p_1$ and Y liters of an acid with $pH = p2$, then the concentration of H^+ ions is

$$\frac{X\,10^{-p_1} + Y\,10^{-p_2}}{X + Y} \qquad\qquad (C.2.18)$$

and the pH of the resulting solution is the negative log of this quantity. So, as long as both the effluent and the receiving waters are both acidic or both basic, the concentration of ions can be considered as a conservative constituent.

The standards for pH require that pH lie between two values: one above 7, the other below. The damage as measured by pH and the distributions of effluent pH can also be divided into two parts: one for $pH > 7$, the other for $pH < 7$. Similarly, to consider the worse case problem, the receiving waters will be assumed to have the same quality (acidic or basic) as the effluent.

The self-monitoring data for pH will either be (1) a monthly maximum and minimum or (2) a monthly maximum, minimum, and mean. If the data are the former, then the mean and standard deviation can be estimated using the midrange and the range respectively. If they are the latter, then two statistical descriptions can be obtained, one using the mean and maximum, the other the mean and minimum. Two standard deviations would be estimated

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* We are assuming that no chemical reaction or buffering takes place.

using the estimation technique described in Appendix A.1. The probability density function for pH would have the shape shown in Figure C.2.1.
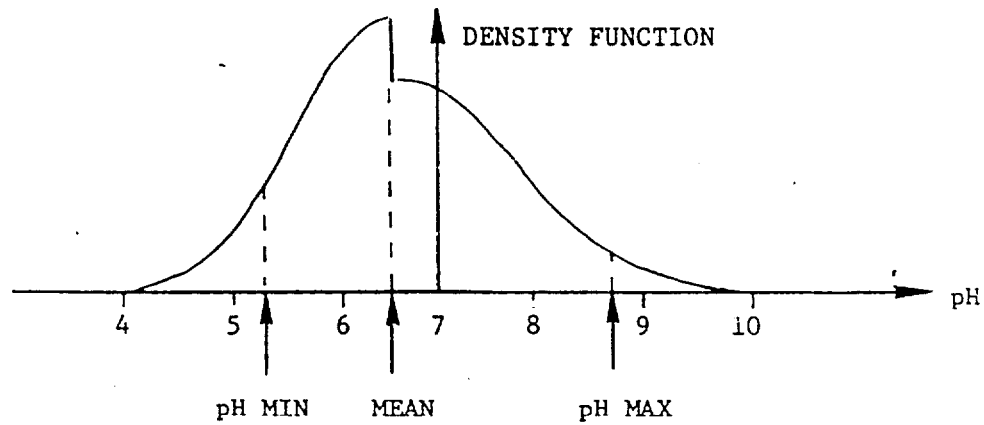

Figure C.2.1   Example of probability density function of pH.

Inputs

The data needed to calculate the expected damage and probability of violation are given below. The subscript J denotes either H or OH. The distribution of pH or pOH is assumed normal.

For source i:

μ_{iJ} = mean of pJ ($\mu_{iOH} = 14 - \mu_{iH}$)

σ_{iJ} = standard deviation of pJ

CU_{iJ} = upstream concentration of J ions (Moles/l)

$EFST_{iJ}$ = effluent standard for pJ

QU_i = flow of stream upstream from source (Ml/day)

QS_i = effluent flow (Ml/day).

The damage function for pH was given in Table 6.1. It is much easier to obtain expressions for the expected damage if the damage function is given in units of concentration of ions. The damage

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function is therefore redefined as shown in Table C.2.1 (the damage function is assumed linear, in concentration, between the given values) and it is plotted in Figure C.2.2. Therefore, for J = H or OH, the following is defined:

$$d_J(k) = \text{concentration of } J \text{ ions when damage equals k-1}$$
$$\text{where } k = 1, 2, ..., 11.$$

The damage function $D_J(\alpha)$

$$D_J(\alpha) = \sum_{k=1}^{10} \left\{ \frac{(\alpha - d_J(k))}{(d_J(k+1) - d_J(k))} + (k-1) \right\} \phi(d(k), d(k+1),$$

$$+ 10 \quad \phi(d(11), \infty, \alpha)$$

(C.2.19)

where ϕ is defined in (C.2.2).

Maximum Downstream Concentration

The maximum downstream concentration of H or OH ions is

$$CO_{iJ} = \frac{CS_{iJ} \; QS_i + CU_{iJ} \; QU_i}{QS_i + QU_i}$$

(C.2.20)

where CS_{iJ} is the concentration of J ions in the effluent. Note that

$$CS_{iJ} = 10^{-pJ}$$

(C.2.21)

where pJ is the pH or pOH of the effluent and is a normal random variable. The upstream concentration is set to give the desired level of damage under zero source load.

TABLE C.2.1   DAMAGE FUNCTION BREAKPOINTS

Damage function value	H^+ ions		OH^- ions	
	pH	Conc	pOH	Conc
0	7.00	1.00×10^{-7}	7.00	1.00×10^{-7}
1	6.75	1.78×10^{-7}	6.50	3.16×10^{-7}
2	6.50	3.16×10^{-7}	6.00	$1.00 \times 10{-6}$
3	6.25	5.62×10^{-7}	5.80	1.58×10^{-6}
4	6.00	1.00×10^{-6}	5.60	2.51×10^{-6}
5	5.50	3.16×10^{-6}	5.30	$5.01 \times 10{-6}$
6	5.00	1.00×10^{-5}	5.00	$1.00 \times 10{-5}$
7	4.50	3.16×10^{-5}	4.50	$3.16 \times 10{-5}$
8	4.00	1.00×10^{-4}	4.00	$1.00 \times 10{-4}$
9	3.95	1.12×10^{-4}	3.95	$1.12 \times 10{-4}$
10	3.90	1.26×10^{-4}	3.90	$1.26 \times 10{-4}$

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